CVPR 2021 Tutorial

Normalization Techniques in Deep Learning: Methods, Analyses and Applications

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Outline

01. Motivations of Normalization Techniques

02. Introduction of Normalization Methods

03. Analyses of Normalization

04. Applications of Normalization

Outline

Introduction of Normalization Methods

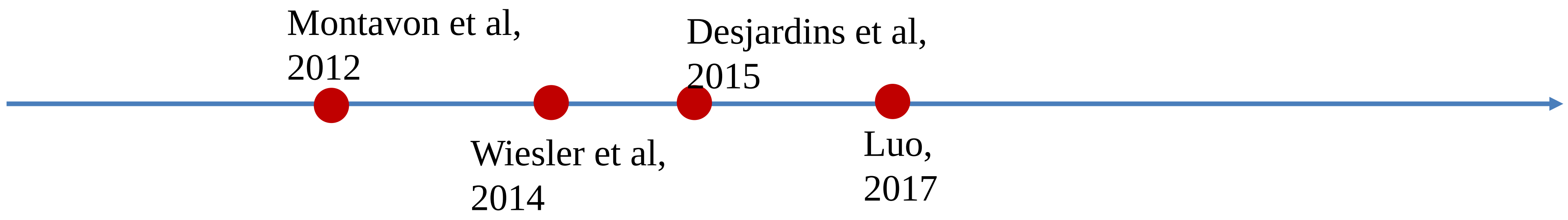
- Normalize Activation
  - By population statistics
  - As functions
- Normalize Weight
- Normalize Gradient

Normalizing activations

- Machine learning/Optimization community:

$$\Sigma_x = \mathbb{E}_{p(x)}\left(xx^{\boldsymbol{T}}\right)$$

Population statistics of a dataset



Normalization by Population Statistics

- Centering the activation
  - Montavon et al, 2014; Wiesler et al, 2014

$$\hat{\mathbf{x}} = \mathbf{x} - \hat{\boldsymbol{\mu}}$$

$\hat{\boldsymbol{\mu}}$ is the mean of activation over the training dataset;
Parameter to be estimated

- Standardizing the activation: centering + scaling
  - Wiesler et al, 2014

$$\hat{\mathbf{x}} = \frac{\mathbf{x} - \hat{\boldsymbol{\mu}}}{\hat{\sigma}}$$

$\hat{\sigma}$ is the standard deviation of activation over the training dataset;
Parameter to be estimated

- Whitening the activations
  - Desjardins et al 2015; Luo, 2017

$$\hat{x}_I = \hat{\Sigma}^{-\frac{1}{2}}(\mathbf{x} - \hat{\boldsymbol{\mu}})$$

$\hat{\Sigma}^{-\frac{1}{2}}$ is the whitening matrix of activation over the training dataset;
Parameter to be estimated

Normalization by Population Statistics

- Advantages
  - Well exploit the beneficial property of normalization in optimization
- Drawbacks
  - Training instability
    - The estimation is not accurate (sampled data)
    - Internal covariant shift (the distribution of activation varying with training progressing)
  - Can not be used to large networks
    - An inaccurate estimation of population statistics will be amplified as the layers increase

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Normalizing activations

- Machine learning/Optimization community:

  Population statistics of a dataset

$$\Sigma_x = \mathbb{E}_{p(x)}(xx^T)$$

Montavon et al, 2012

Wiesler et al, 2014

Desjardins et al, 2015

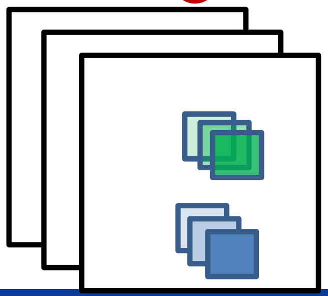
Luo, 2017

- Computer vision community:

  Local statistics in an sample

Jarrett et al, 2009

Krizhevsky et al, 2012



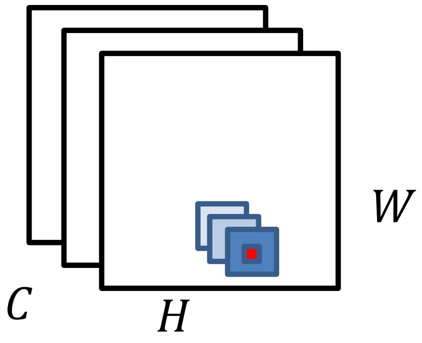
Ren et al, 2017

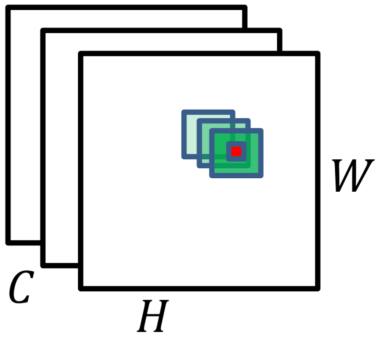
Ortiz et al, 2020

Local Normalization in a Sample

- Local normalization
  - Local contrast normalization [Jarrett et al, ICCV 2009]
  - Local response normalization [Krizhevsky et al, NeurIPS 2012]
  - Divisive normalization [Ren et al, ICLR 2017]
  - Local context normalization [Ortiz et al, CVPR 2020]

Given an example $X \in \mathbb{R}^{C \times H \times M}$





Local Normalization in a Sample

- Advantage
  - Training is somewhat stable due to back-propagating through normalization
  - The visual contrast invariant property may benefit generalization

- Limits
  - Specific to visual data (feature maps)
  - May change the representation ability and reduce the discriminative information
  - It is not clear whether benefits optimization

Normalizing activations

- Machine learning/Optimization community:

  Population statistics of a dataset

$$\Sigma_x = \mathbb{E}_{p(x)}\,(xx^T)$$

Montavon et al, 2012

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Desjardins et al, 2015

Luo, 2017

Batch Normalization, Ioffe and Szegedy, ICML 2015

- Computer Vision Community:

  Local statistics in an sample

Jarrett et al, 2009

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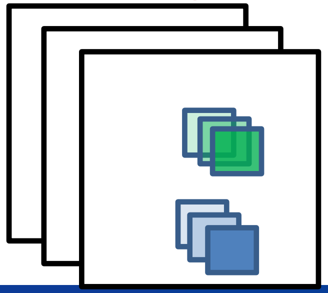

Ren et al, 2017

Ortiz et al, 2020

Batch Normalization (BN)

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1\ldots m}\}$;
Parameters to be learned: γ, β

Output: $\{y_i = \mathrm{BN}_{\gamma,\beta}(x_i)\}$

$$\mu_\mathcal{B} \leftarrow \frac{1}{m}\sum_{i=1}^{m} x_i \qquad \text{// mini-batch mean}$$

$$\sigma_\mathcal{B}^2 \leftarrow \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_\mathcal{B})^2 \qquad \text{// mini-batch variance}$$

$$\widehat{x}_i \leftarrow \frac{x_i - \mu_\mathcal{B}}{\sqrt{\sigma_\mathcal{B}^2 + \epsilon}} \qquad \text{// normalize}$$

$$y_i \leftarrow \gamma\widehat{x}_i + \beta \equiv \mathrm{BN}_{\gamma,\beta}(x_i) \qquad \text{// scale and shift}$$

Normalize over mini-batch data

Extra learnable scale and bias

Back-propagate through the transformation

$$\frac{\partial \ell}{\partial \sigma_\mathcal{B}^2} = \sum_{i=1}^{m} \frac{\partial \ell}{\partial \widehat{x}_i} \cdot (x_i - \mu_\mathcal{B}) \cdot \frac{-1}{2}(\sigma_\mathcal{B}^2 + \epsilon)^{-3/2}$$

$$\frac{\partial \ell}{\partial \mu_\mathcal{B}} = \left(\sum_{i=1}^{m} \frac{\partial \ell}{\partial \widehat{x}_i} \cdot \frac{-1}{\sqrt{\sigma_\mathcal{B}^2+\epsilon}}\right) + \frac{\partial \ell}{\partial \sigma_\mathcal{B}^2} \cdot \frac{\sum_{i=1}^{m} -2(x_i-\mu_\mathcal{B})}{m}$$

$$\frac{\partial \ell}{\partial x_i} = \frac{\partial \ell}{\partial \widehat{x}_i} \cdot \frac{1}{\sqrt{\sigma_\mathcal{B}^2+\epsilon}} + \frac{\partial \ell}{\partial \sigma_\mathcal{B}^2} \cdot \frac{2(x_i-\mu_\mathcal{B})}{m} + \frac{\partial \ell}{\partial \mu_\mathcal{B}} \cdot \frac{1}{m}$$

Batch normalization accelerating deep network training by reducing internal covariate shift[Ioffe and Szegedy, ICML 2015]

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Normalize over mini-batch data

mini-batch statistics for training and population statistics for inference

$$\begin{cases} \hat{u} = (1-\lambda)\hat{u} + \lambda u, \\ \hat{\sigma}^2 = (1-\lambda)\hat{\sigma}^2 + \lambda\sigma^2 \end{cases}$$

Back-propagate through the transformation

$$\frac{\partial \ell}{\partial \sigma_{\mathcal{B}}^2} = \sum_{i=1}^{m} \frac{\partial \ell}{\partial \widehat{x}_i} \cdot (x_i - \mu_{\mathcal{B}}) \cdot \frac{-1}{2}(\sigma_{\mathcal{B}}^2 + \epsilon)^{-3/2}$$

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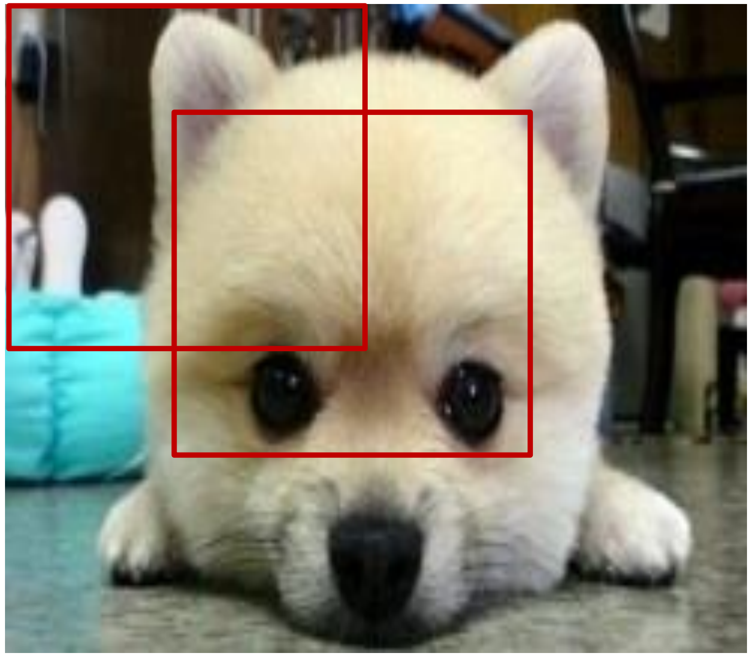
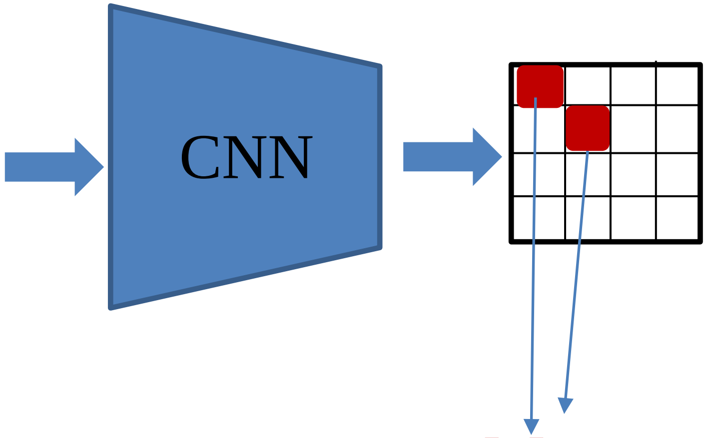
Extra learnable scale and bias

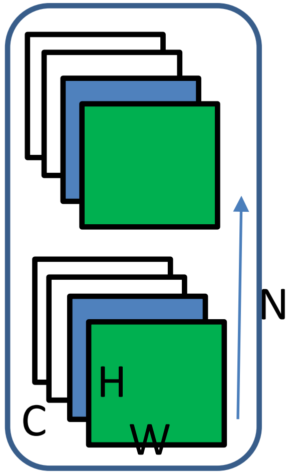
Batch normalization accelerating deep network training by reducing internal covariate shift[Ioffe and Szegedy, ICML 2015]

Batch Normalization (BN)

- Extension to CNN input
  - Jarrett et al, 2009; Gulcehre and Bengio, 2013





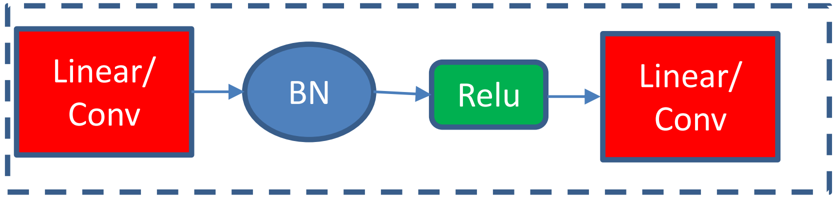


$\mathbf{X} \in \mathcal{R}^{N \times C \times H \times W}$

Spatial location as an example for normalization

Batch normalization accelerating deep network training by reducing internal covariate shift[Ioffe and Szegedy, ICML 2015]

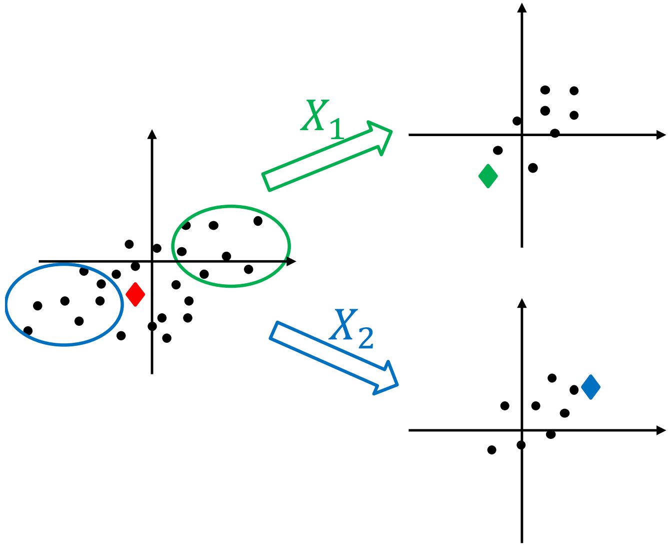
Batch Normalization: Property



- For accelerating training
  - Stable training
    - Weight scale invariant: not sensitive for weight initialization

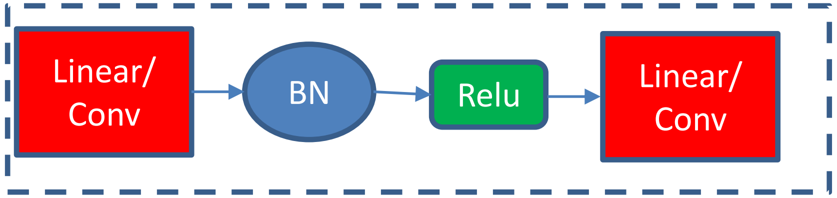
$$\mathrm{BN}(W\mathrm{u}) = \mathrm{BN}((aW)\mathrm{u})$$

  - Better conditioning
    - Can use large learning rate



- For generalization
  - Introduced stochasticity
    - Mini-batch dependence during training
    - Training-test discrepancy
  - Scale invariant representation

Batch Normalization: Property



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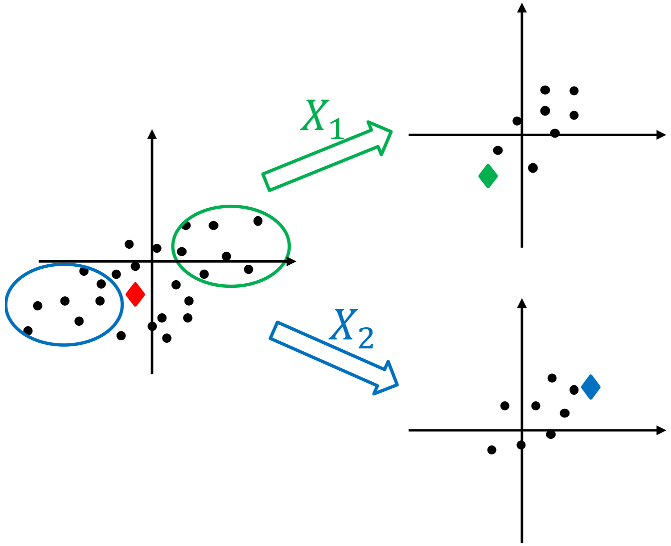


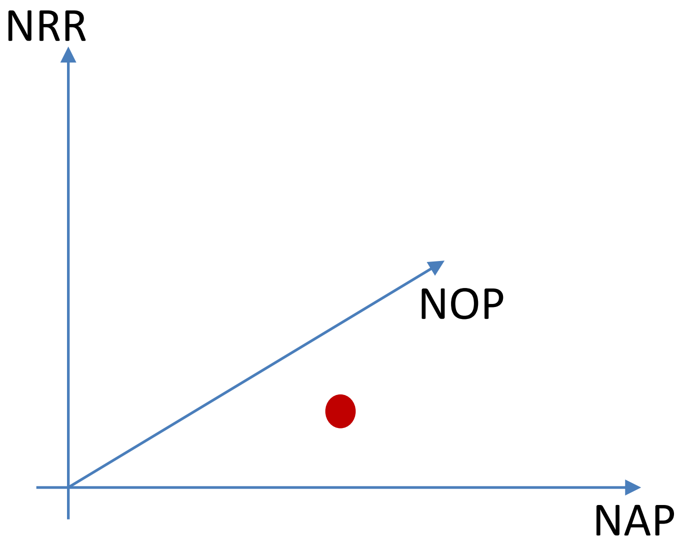
- Independent population statistics or sharing statistics in RNN?
- Data are from different domains
- Small batch size problem

Outline

- Normalizing activations as functions
  - A Framework for decomposing normalization
  - Multi-Mode and combinational normalization
  - BN for more robust estimation

A Framework for Decomposing Normalization

- The framework
  - Normalization Area Partitioning (NAP): which area to calculate the ‘statistics’
  - Normalization Operation (NOP): what kind of normalization operation?
  - Normalization Representation Recovery (NRR)



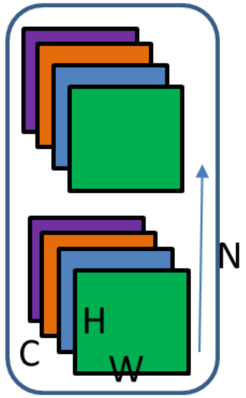
Algorithm 1 Framework of algorithms normalizing activations as functions.

1: **Input**: mini-batch inputs $\mathbf{X} \in \mathbb{R}^{d\times m\times h\times w}$.
2: **Output**: $\widetilde{\mathbf{X}} \in \mathbb{R}^{d\times m\times h\times w}$.
3: Normalization area partitioning: $\boldsymbol{X} = \Pi(\mathbf{X})$.
4: Normalization operation: $\widehat{\boldsymbol{X}} = \Phi(\boldsymbol{X})$.
5: Normalization representation recovery: $\widetilde{\boldsymbol{X}} = \Psi(\widehat{\boldsymbol{X}})$.
6: Reshape back: $\widetilde{\mathbf{X}} = \Pi^{-1}(\widetilde{\boldsymbol{X}})$.

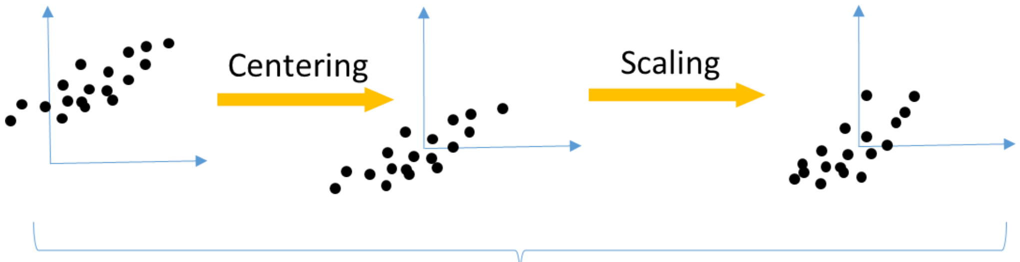
A Framework for Decomposing Normalization

- Batch Normalization
  - NAP: $\boldsymbol{X} = \Pi_{BN}(\mathbf{X}) \in \mathbb{R}^{d\times mhw}$

$\mathbf{X} \in \mathcal{R}^{N\times C\times H\times W}$



  - NOP: $\widehat{\boldsymbol{X}} = \Phi_{SD}(\boldsymbol{X}) = \Lambda^{-\frac{1}{2}}(\boldsymbol{X} - \mathbf{u}\mathbf{1}^T)$.



  - NRR: $\widetilde{\boldsymbol{X}} = \Psi_{AF}(\widehat{\boldsymbol{X}}) = \widehat{\boldsymbol{X}} \odot (\gamma\mathbf{1}^T) + (\beta\mathbf{1}^T)$

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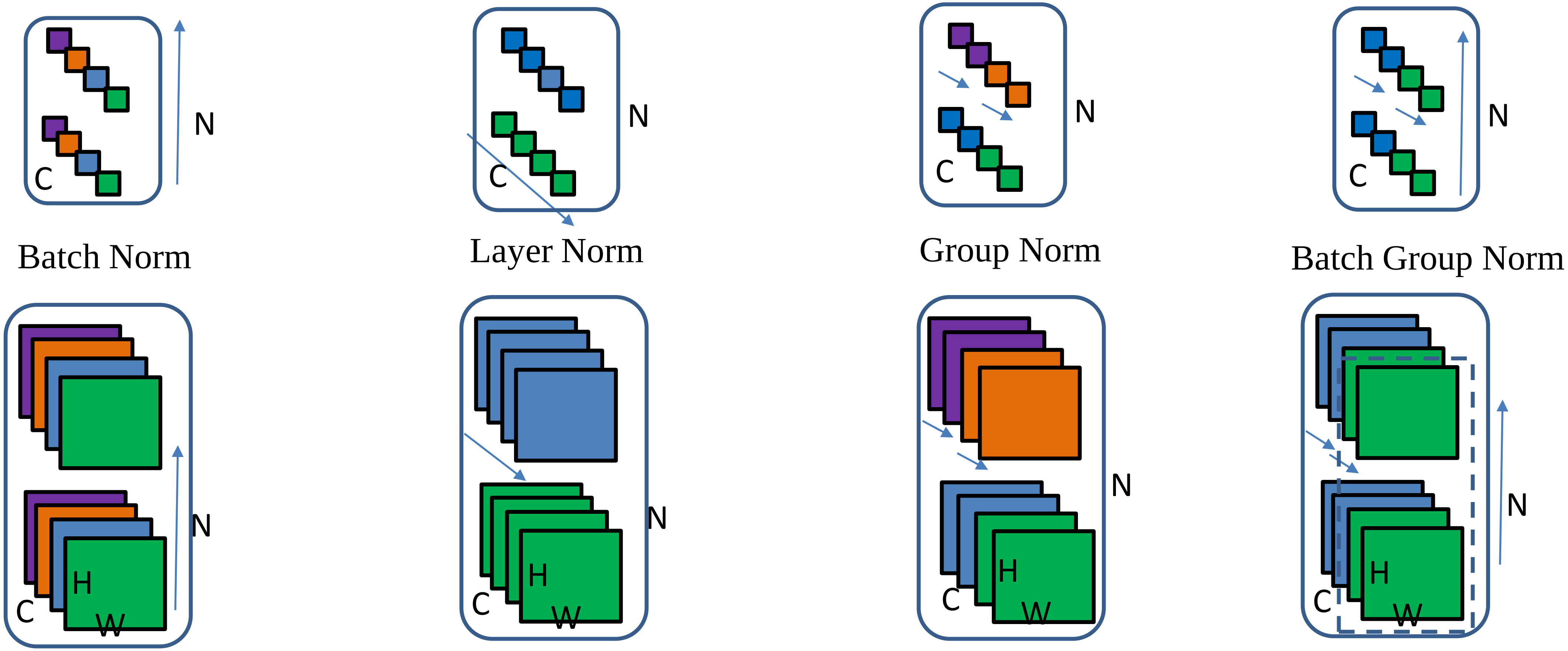
Normalization Area Partitioning

- MLP input: $X \in \mathcal{R}^{N \times C}$

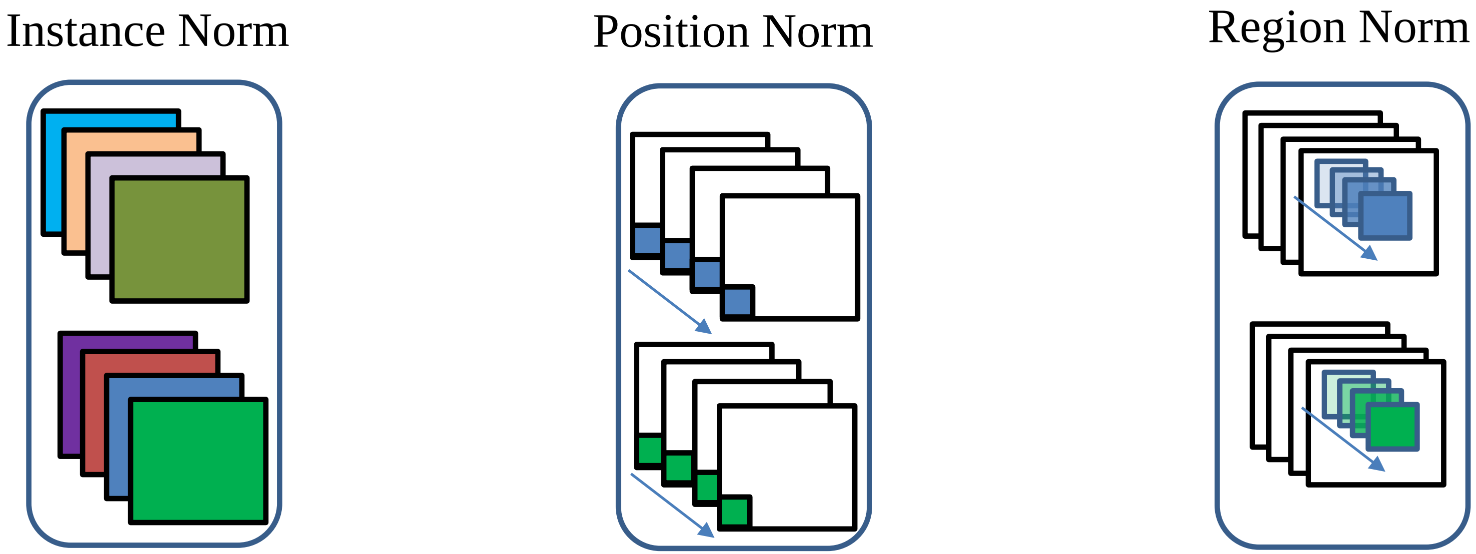


- CNN input: $\mathbf{X} \in \mathcal{R}^{N \times C \times H \times W}$

[Ioffe and Szegedy, ICML 2015; Ba et al, 2016; Wu and He, ECCV 2018; Summers and Dinneen, ICLR 2020]

Normalization Area Partitioning

- CNN Input: $X \in \mathcal{R}^{N\times C\times H\times W}$



[Ulyanov et al, 2016; Ba et al, 2016; Li et al, NeurIPS 2019; Yu et al, AAAI 2020]

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Normalization Operation

- Batch Whitening (BW)

Activation distribution

Standardization:

$$\hat{X} = \varphi(X) = \left(diag(\Sigma)\right)^{-\frac{1}{2}}(X - \mu 1^T)$$

$$diag(\hat{X}\hat{X}^T) = I$$

Covariance $matrix$

Whitening:

$$\hat{X} = \varphi(X) = \Sigma^{-\frac{1}{2}}(X - \mu 1^T)$$

$$\hat{X}\hat{X}^T = I$$

Standardization is a special case of whitening

Whitening further improver conditioning over standardization

Decorrelated Batch Normalization [Huang et al, CVPR 2018]

Normalization Operation

- Batch Whitening (BW)

➢ Forward

$$\mu = \frac{1}{m}\sum_{j=1}^{m}\mathbf{x}_j$$

$$\Sigma = \frac{1}{m}\sum_{j=1}^{m}(\mathbf{x}_j-\mu)(\mathbf{x}_j-\mu)^T$$

$$\Sigma = \mathbf{D}\Lambda\mathbf{D}^T$$

$$\mathbf{U} = \Lambda^{-1/2}\mathbf{D}^T$$

$$\tilde{\mathbf{x}}_i = \mathbf{U}(\mathbf{x}_i-\mu)$$

$$\hat{\mathbf{x}}_i = \mathbf{D}\tilde{\mathbf{x}}_i$$

➢ Backward

$$\frac{\partial L}{\partial \tilde{\mathbf{x}}_i} = \frac{\partial L}{\partial \hat{\mathbf{x}}_i}\mathbf{D}$$

$$\frac{\partial L}{\partial \mathbf{U}} = \sum_{i=1}^{m}\frac{\partial L}{\partial \tilde{\mathbf{x}}_i}^T(\mathbf{x}_i-\mu)^T$$

$$\frac{\partial L}{\partial \Lambda} = (\frac{\partial L}{\partial \mathbf{U}})\mathbf{D}(-\frac{1}{2}\Lambda^{-3/2})$$

$$\frac{\partial L}{\partial \mathbf{D}} = \frac{\partial L}{\partial \mathbf{U}}^T\Lambda^{-1/2} + \sum_{i=1}^{m}\frac{\partial L}{\partial \hat{\mathbf{x}}_i}^T\tilde{\mathbf{x}}_i^T$$

[Ionescu et al, ICCV 2015]

$$\boxed{\frac{\partial L}{\partial \Sigma} = \mathbf{D}\{(\mathbf{K}^T\odot(\mathbf{D}^T\frac{\partial L}{\partial \mathbf{D}})) + (\frac{\partial L}{\partial \Lambda})_{diag}\}\mathbf{D}^T}$$

$$\frac{\partial L}{\partial \mu} = \sum_{i=1}^{m}\frac{\partial L}{\partial \tilde{\mathbf{x}}_i}(-\mathbf{U}) + \sum_{i=1}^{m}\frac{-2(\mathbf{x}_i-\mu)^T}{m}(\frac{\partial L}{\partial \Sigma})_{sym}$$

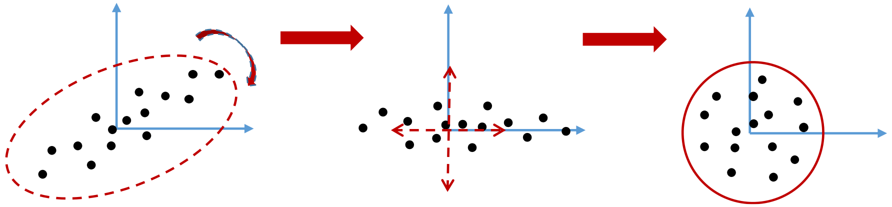
$$\frac{\partial L}{\partial \mathbf{x}_i} = \frac{\partial L}{\partial \tilde{\mathbf{x}}_i}\mathbf{U} + \frac{2(\mathbf{x}_i-\mu)^T}{m}(\frac{\partial L}{\partial \Sigma})_{sym} + \frac{1}{m}\frac{\partial L}{\partial \mu}$$

Decorrelated Batch Normalization [Huang et al, CVPR 2018]

Batch Whitening

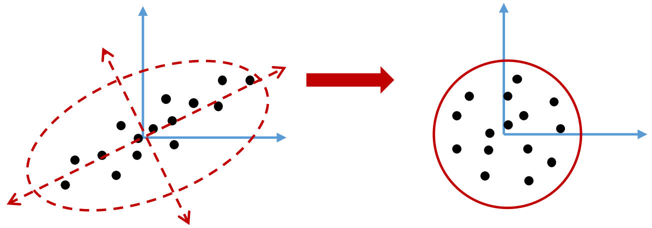
- Whitening
  - PCA whitening not work

$$G_{PCA} = \Lambda^{-\frac{1}{2}} D^T, \qquad D\Lambda D^T = \Sigma$$

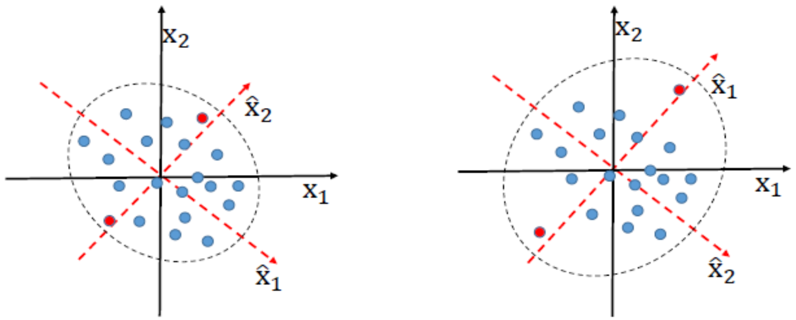


  - ZCA whitening work

$$G_{ZCA} = D\Lambda^{-\frac{1}{2}} D^T$$

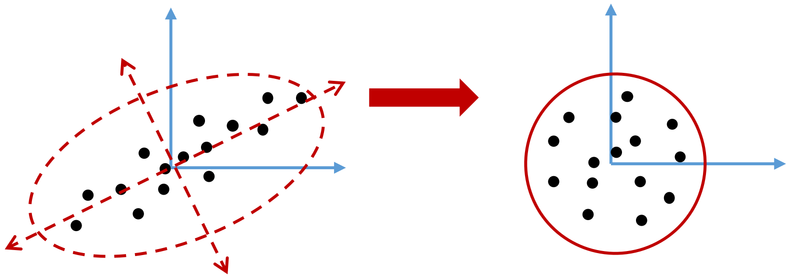


- PCA cause stochastic axis swapping

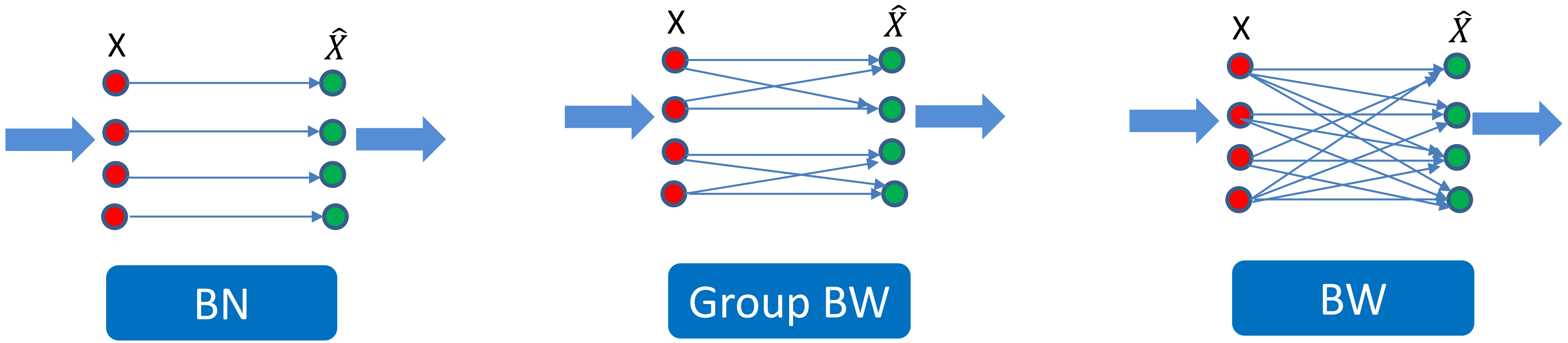


Control the Extent of Batch Whitening

- What's the problem of full whitening
  - Computational cost
  - Overmuch constraints on $\hat{X}$ ($\hat{X}\hat{X}^T = I$)
  - Difficulty in estimating the population statistics



- Group based batch whitening



Control the Extent of Batch Whitening

- Newton's iteration to calculate whitening matrix

$$\hat{X} = \varphi(X) = \Sigma^{-\frac{1}{2}}(X - \mu 1^T)$$

Normalize eigenvalues: $\Sigma_N = \Sigma / tr(\Sigma)$

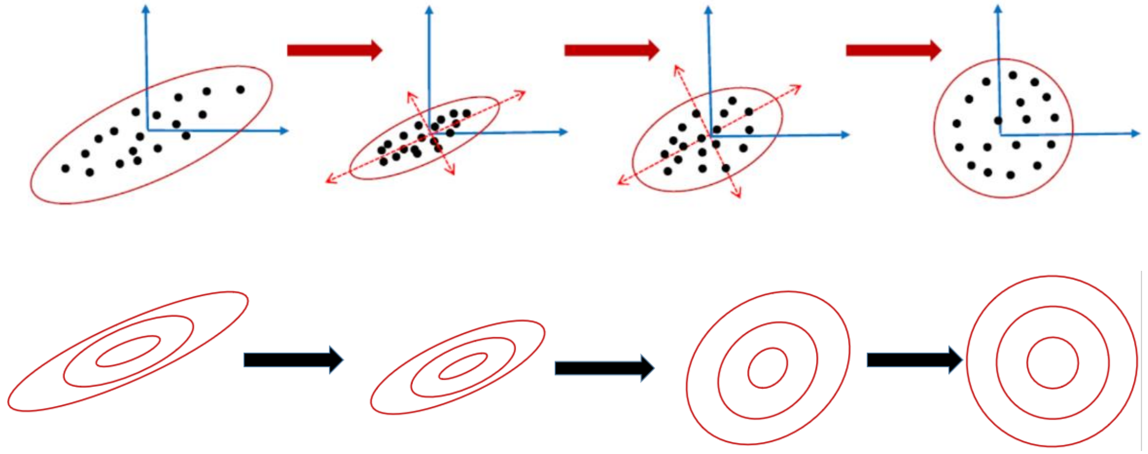
Iteration:

$\mathbf{P}_0 = \mathbf{I}.$
for $k = 1$ to T **do**
$\mathbf{P}_k = \frac{1}{2}(3\mathbf{P}_{k-1} - \mathbf{P}_{k-1}^3 \Sigma_N)$
end for

Whitening matrix: $\Sigma^{-\frac{1}{2}} = \mathbf{P}_T / \sqrt{tr(\Sigma)}$

Activation distribution

Optimization landscape



Iterative Normalization: Beyond Standardization towards Efficient Whitening [Huang et al, CVPR 2019]

Batch Whitening

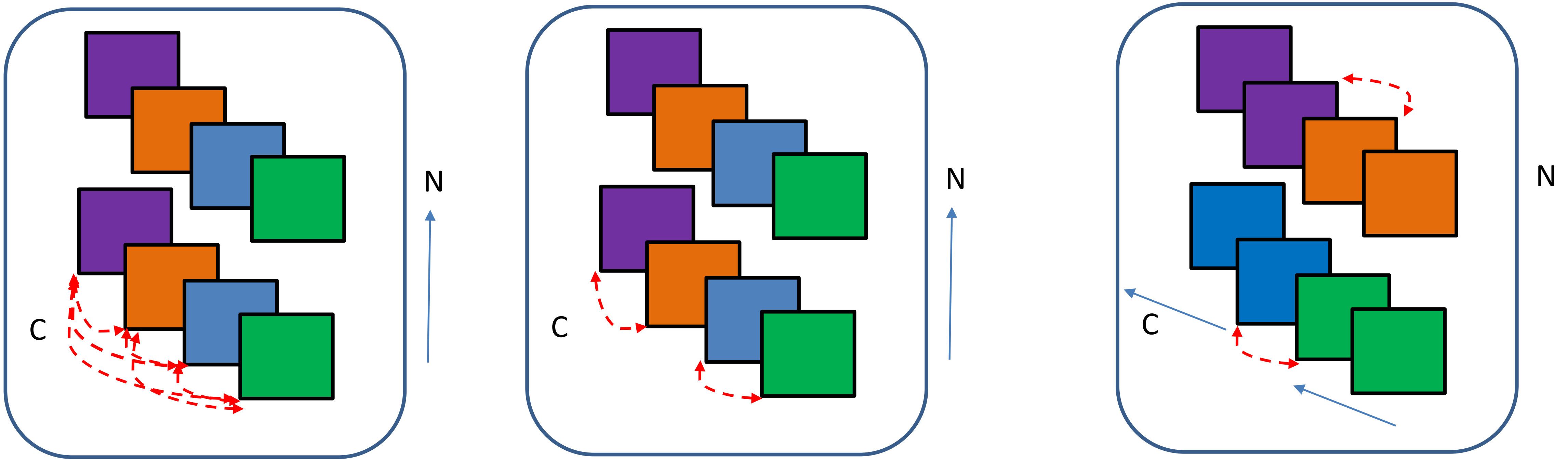
- Advantages over standardization
  - Better conditioning theoretically
  - Probably better generalization (the amplified stochasticity) by controlling the extent of whitening

- Disadvantages
  - Computational costs ⟹ Group based, Newtown's iteration
  - Numerical instability ⟹ Cholesky decomposition, Newtown's iteration
  - More difficulty in ensuring the training and inference consistency

[Huang et al CVPR 2018; Siarohin et al ICLR 2019; Huang et al CVPR 2019; Ye et al ICLR 2020; Huang et al, CVPR 2020]

Group whitening

- Exploiting the advantages of whitening and avoid the disadvantages of normalization over batch



BW | Group based BW | Group Whitening

Group Whitening: Balancing Learning Efficiency and Representational Capacity [Huang et al, CVPR 2021]

Normalization Operation

- Variations of standardization

$$\hat{x}^{(i)} = \frac{x^{(i)} - u}{\sqrt{\sigma^2 + \epsilon}}$$

  – L^2 Norm (BN): $\sigma^2 = \frac{1}{m}\sum_{i=1}^{m}(x^{(i)} - u)^2$

  – L^1 Norm : $\sigma = \frac{1}{m}\sum_{i=1}^{m}|x^{(i)} - u|$

  – L^∞ Norm : $\sigma = \max_i |x^{(i)}|$

  – More general L^p: $\sigma = \frac{1}{m}\sqrt[p]{\sum_{i=1}^{m}(x^{(i)})^p}$

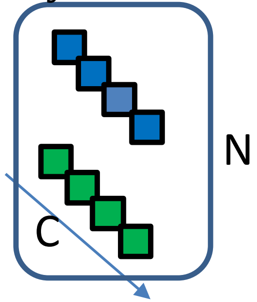
$\left.\begin{matrix} L^1 \\ L^\infty \end{matrix}\right\}$ more efficient and numerical stability in a low precision implementation

[Liao et al, 2016; Wu et al, 2018; Hoffer et al, 2018]

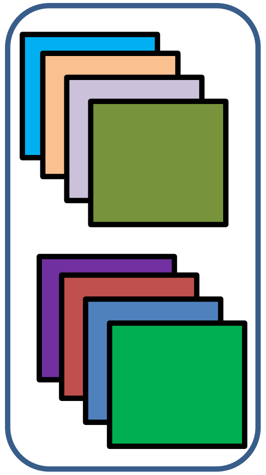
Normalization Operation

- Reduced standardization
  - Centering only (Mean only BN)
  - Scaling only

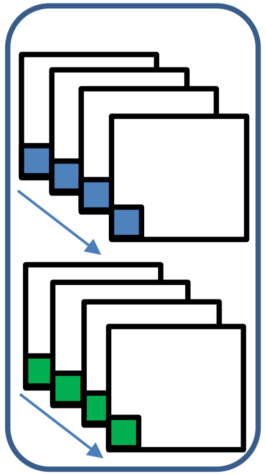
Root Mean Square Layer Norm



Filter Response Normalization



Pixel Normalization



Standardization: Layer Normalization | Instance Normalization | Position Normalization

[Karras, et al, ICLR 2018; Zhang and Sennrich, NeurIPS 2019; Singh and Krishnan, CVPR2020et al, 2018]

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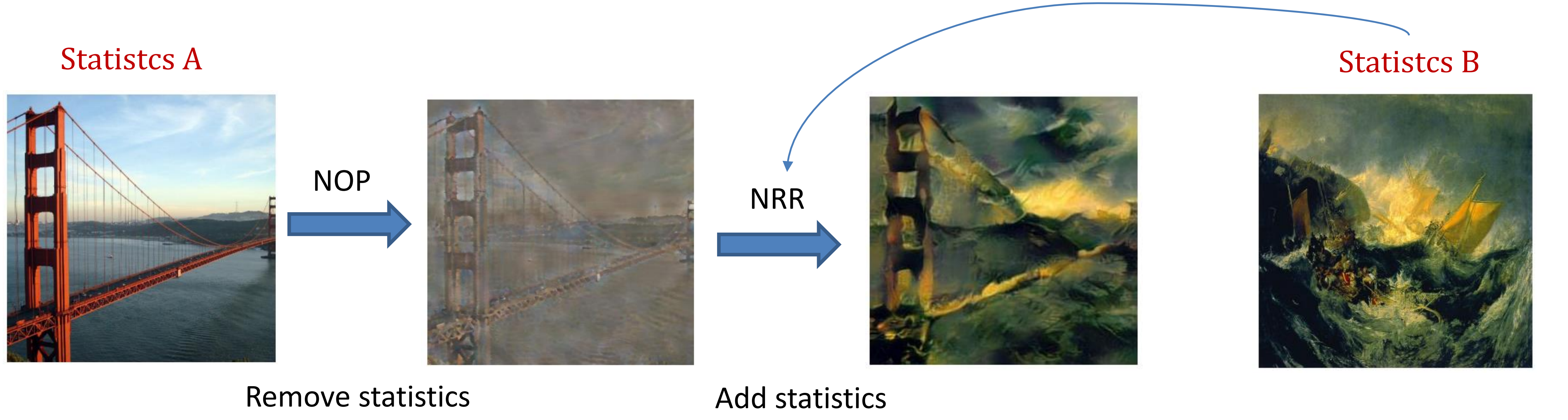
Normalization Representation Recovery

- Why NRR
  - Recover the representation
  - Edit the statistical distribution

NOP: $\widehat{\boldsymbol{X}} = \Phi_{SD}(\boldsymbol{X}) = \Lambda^{-\frac{1}{2}}(\boldsymbol{X} - \mathbf{u}\mathbf{1}^T)$

NRR: $\widetilde{\boldsymbol{X}} = \Psi_{AF}(\widehat{\boldsymbol{X}}) = \widehat{\boldsymbol{X}} \odot (\gamma\mathbf{1}^T) + (\beta\mathbf{1}^T)$

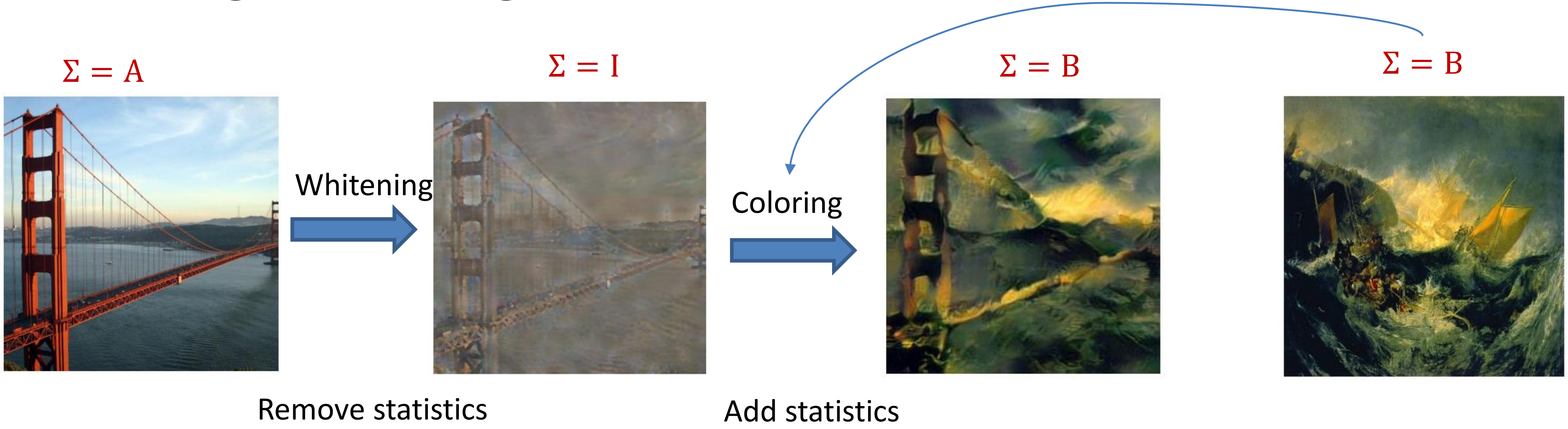


Normalization Representation Recovery

- More general coloring transformation

$$\widetilde{\boldsymbol{X}} = \Psi_{LR}(\widehat{\boldsymbol{X}}) = \widehat{\boldsymbol{X}}\mathbf{W} + (\beta\mathbf{1}^T)$$

- Whitening + Coloring



Dynamic Generate NRR

- Dynamic generate the affine parameters

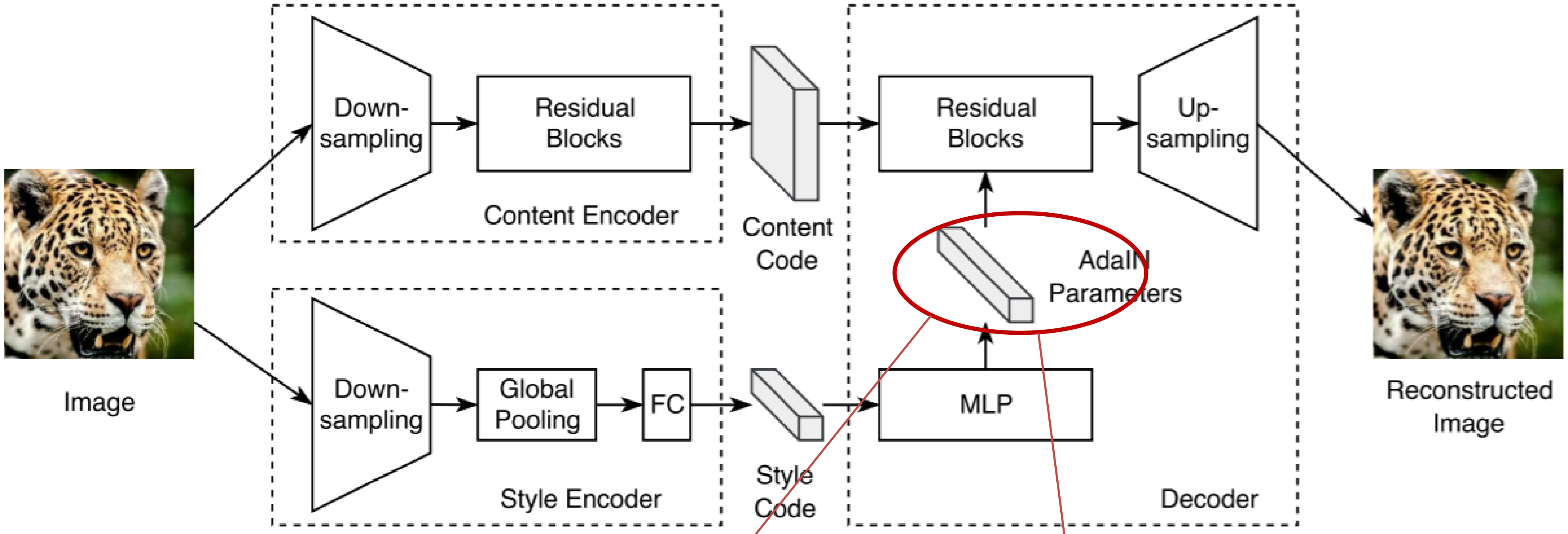
$$\widetilde{\boldsymbol{X}} = \Psi_{DC}(\widehat{\boldsymbol{X}}) = \widehat{\boldsymbol{X}} \odot \Gamma_{\phi^\gamma} + B_{\phi^\beta}$$

where $\Gamma_{\phi^\gamma} \in \mathbb{R}^{d\times m}$ and $B_{\phi^\beta} \in \mathbb{R}^{d\times m}$ are generated by the subnetworks $\phi^\gamma_{\theta_\gamma}(\cdot)$ and $\phi^\beta_{\theta_\beta}(\cdot)$, respectively.

  – Dynamic layer normalization [Kim et al, 2017]

Dynamic Generate NRR

- Adaptive instance normalization

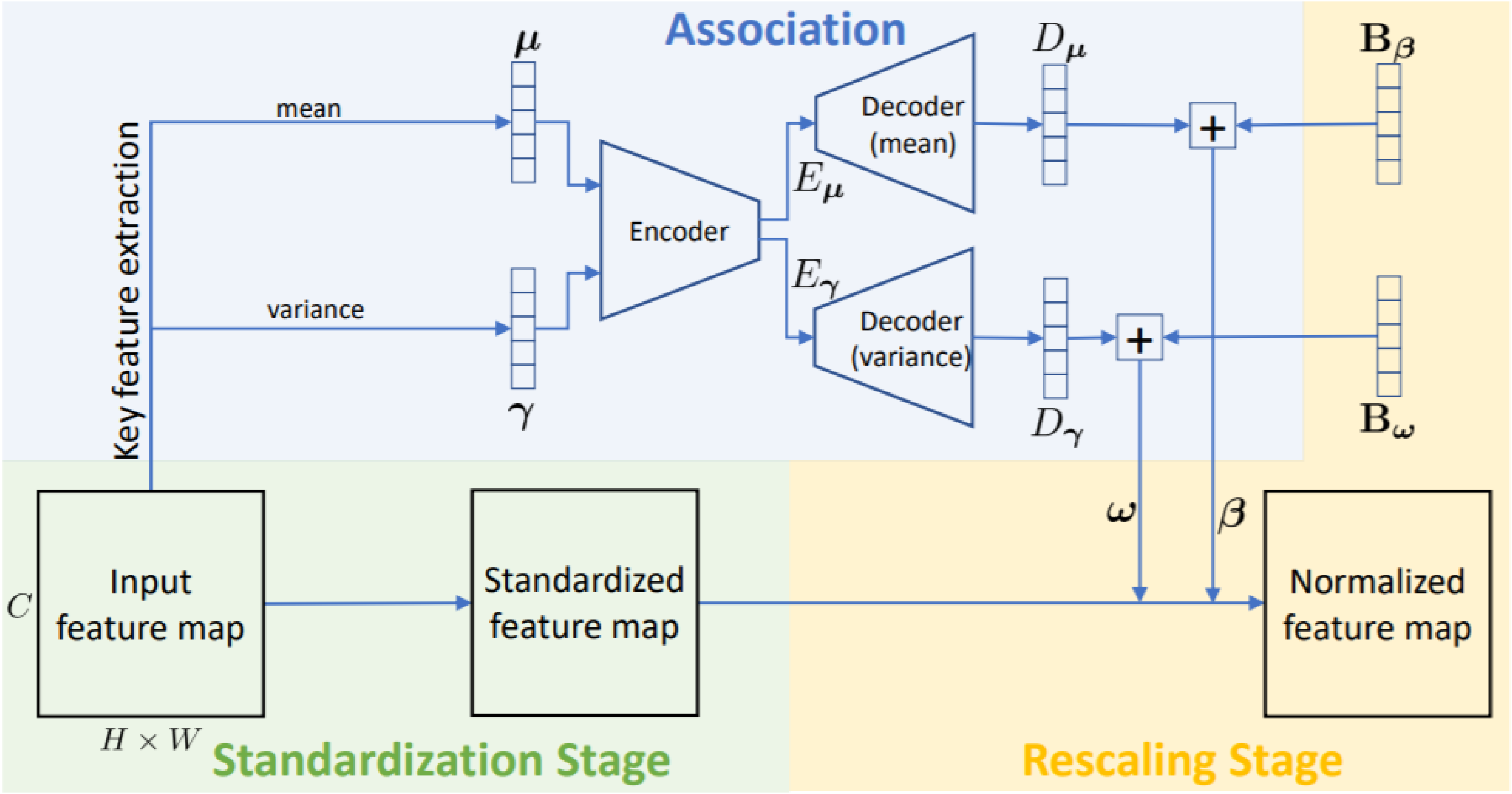


$$\mathrm{AdaIN}(z,\gamma,\beta)=\gamma\left(\frac{z-\mu(z)}{\sigma(z)}\right)+\beta$$

Multimodal Unsupervised Image-to-Image Translation [Huang et al, ECCV 2018]

Dynamic Generate NRR

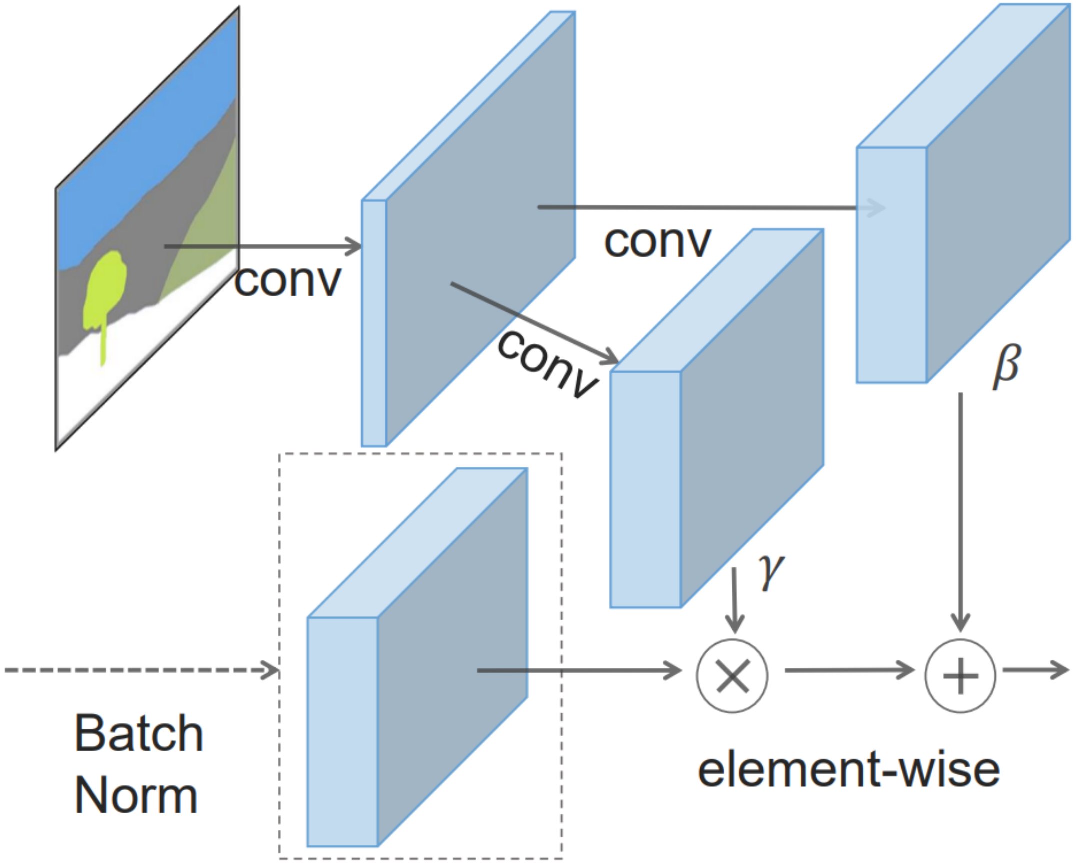
- Instance level-meta norm



Instance-Level Meta Normalization [Jia et al, CVPR 2019]

Dynamic Generate NRR

- Spatially Adaptive Denormalization (SPADE)

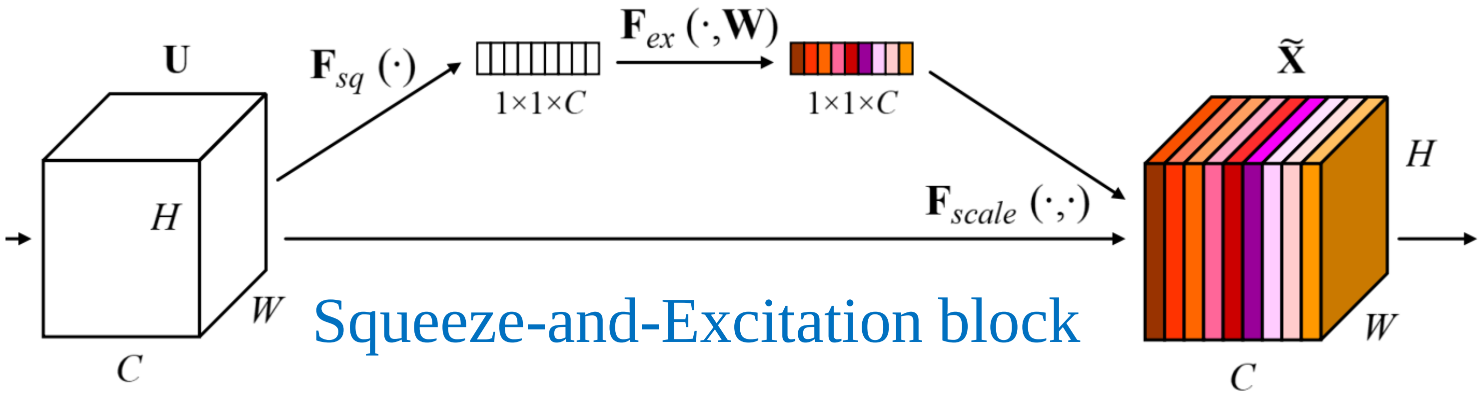


$$\beta, \gamma \in \mathbb{R}^{d \times h \times w}$$

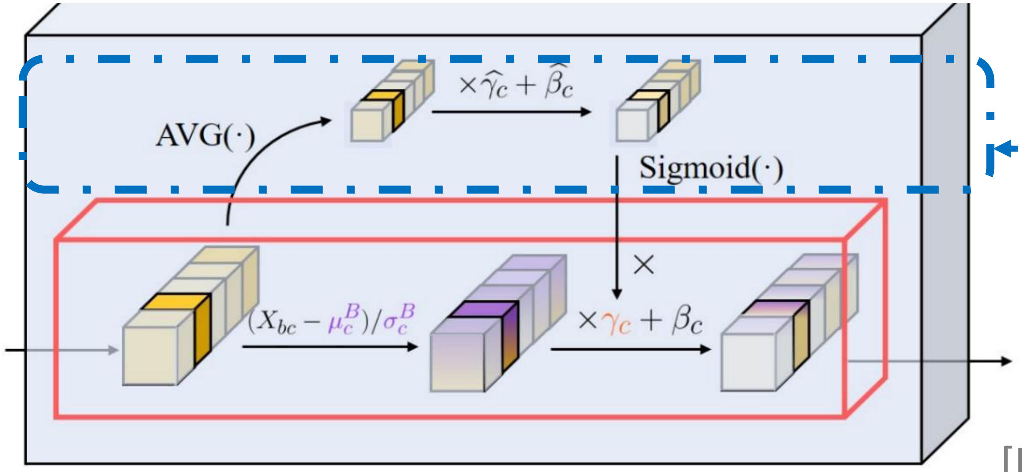
Semantic Image Synthesis with Spatially-Adaptive Normalization [Park et al, CVPR 2019]

Dynamic Generate NRR

- The explanation of channel attention
  - Instance Enhancement Batch Normalization (IEBN)



Squeeze-and-Excitation block

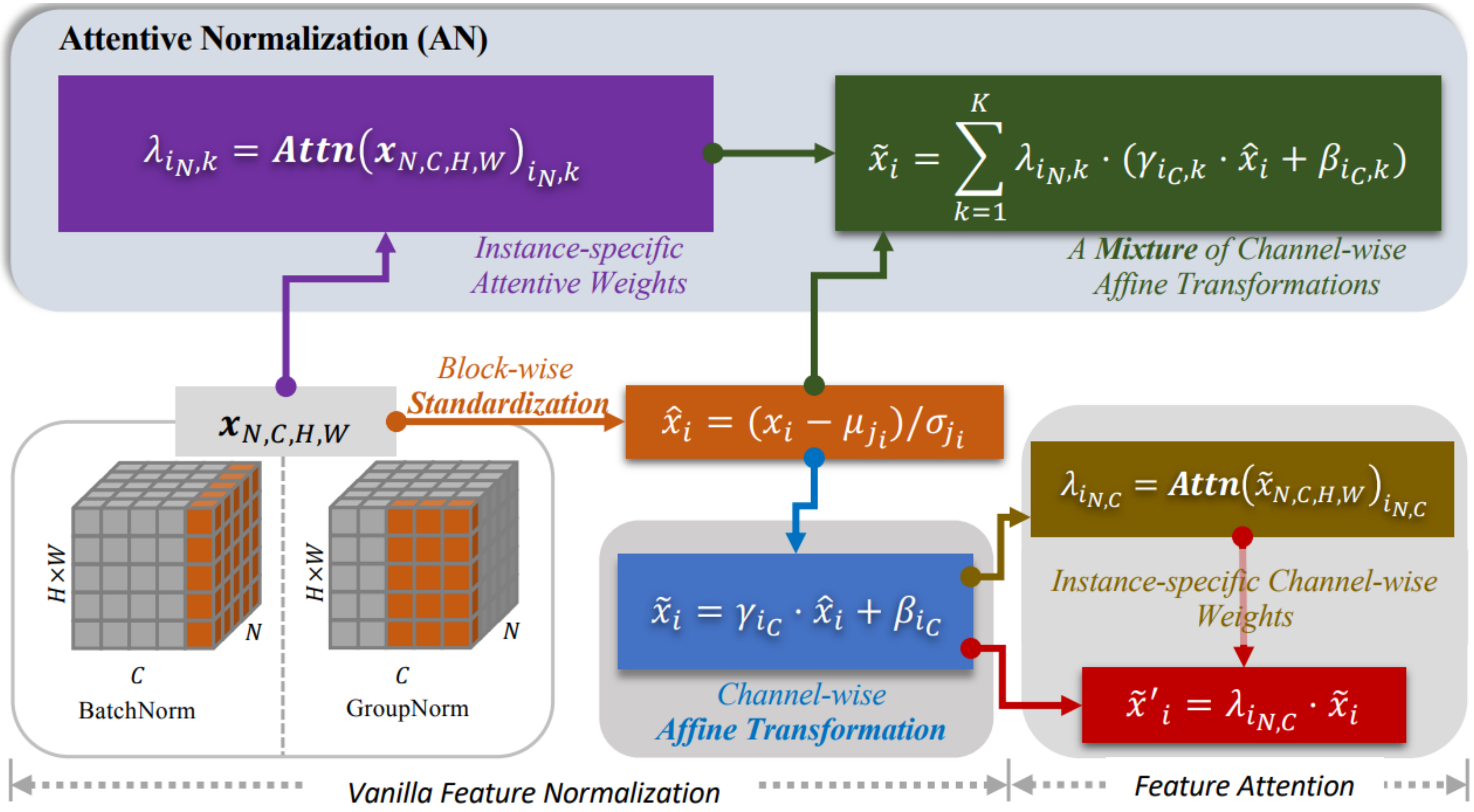


Channel attention

[Hu et al, CVPR 2018; Liang et al, AAAI 2020]

Dynamic Generate NRR

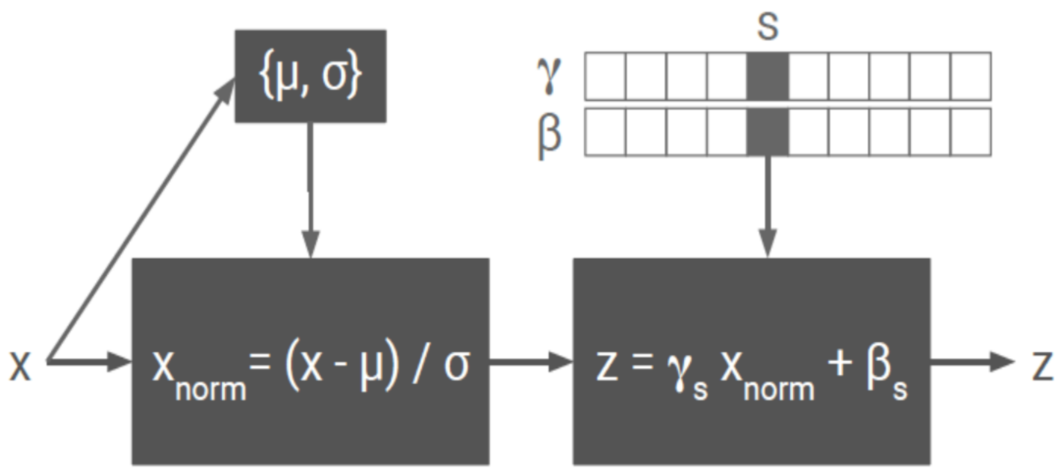
- Attentive Normalization (AN)

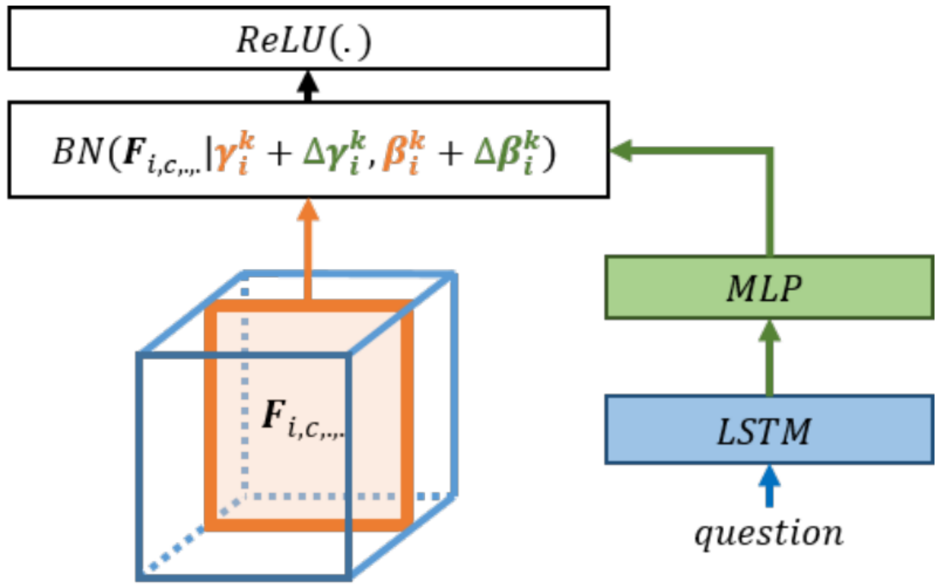


Attentive Normalization [Li et al, ECCV 2020]

Side Information NRR

- Conditional instance normalization
- Conditional batch normalization





A Learned Representation For Artistic Style [Dumoulin et al, ICLR 2017)]

Modulating early visual processing by language [Vries et al, NeurIPS 2017]

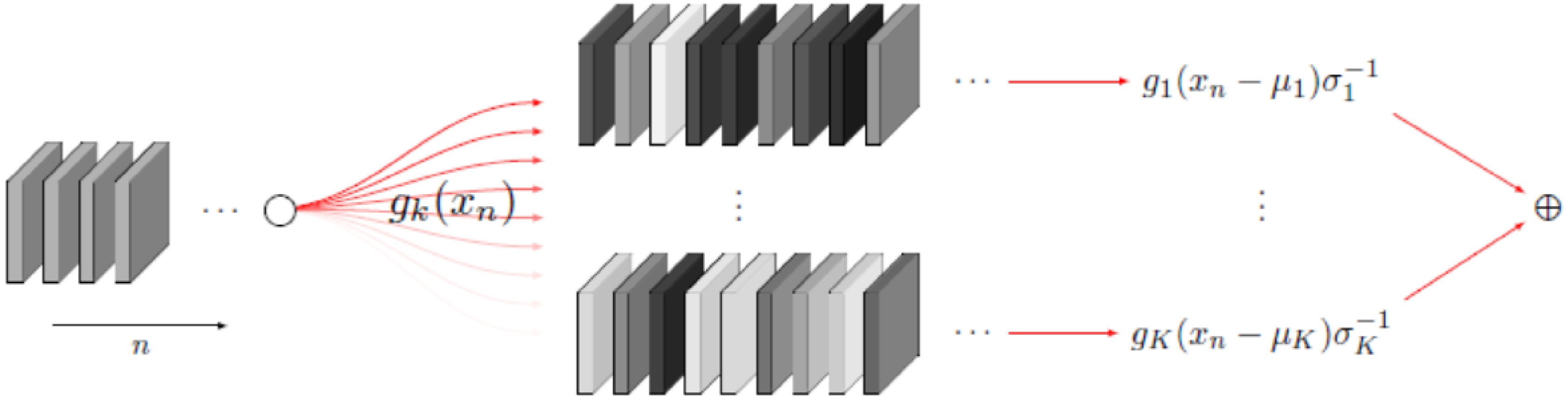
Outline

- Normalizing activations as functions
  - A Framework for decomposing normalization
    - Normalization Area Partitioning (NAP)
    - Normalization Operation (NOP)
    - Normalization Representation Recovery (NRR)
  - Multi-Mode and combinational normalization
  - BN for more robust estimation

Multi-mode Normalization

- Mode normalization

$$\mathrm{MN}(x_n) \triangleq \alpha\left(\sum_{k=1}^{K} g_k(x_n)\frac{x_n-\mu_k}{\sigma_k}\right)+\beta$$



Mode Normalization [Deecke et al, ICLR 2019]

Combinational Normalization

- Switchable Normalization (SN)
  - Combing BN, LN and IN

$$\hat{x}_{nchw} = \gamma \frac{x_{nchw} - (w_{IN}\mu_{IN} + w_{BN}\mu_{BN} + w_{LN}\mu_{LN})}{\sqrt{w'_{IN}\sigma^2_{IN} + w'_{BN}\sigma^2_{BN} + w'_{LN}\sigma^2_{LN}}} + \beta$$

$$w_k = \frac{e^{\lambda_k}}{\sum_{z\in\{IN,LN,BN\}} e^{\lambda_z}}, k \in \{IN, LN, BN\}$$

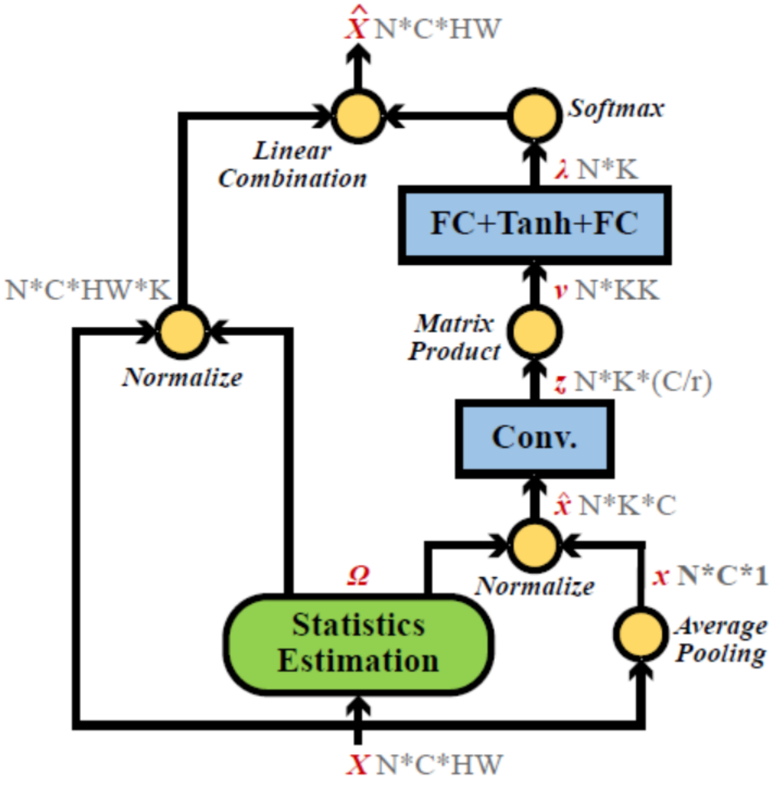
- Switchable Whitening (SW)
  - $k \in \{BW, IW\}$ or $k \in \{BW, IW, IN, LN, BN\}$

[Luo et al, ICLR 2018; Pan et al, ICCV 2019]

Combinational Normalization

- Exemplar Normalization

$$\widehat{X}_n = \sum_k \gamma^k (\lambda_n^k \frac{X_n - \mu^k}{\sqrt{(\delta^k)^2 + \epsilon}}) + \beta^k$$



- Representative Batch Normalization
  - During training: Mini-batch statistics+ instance statistics
  - During inference: Population statistics + instance statistics

[Zhang et al, CVPR 2020; Gao et al, CVPR 2021]

Combinational Normalization

- Batch-Instance Normalization

$$\mathbf{y} = \left(\rho \cdot \hat{\mathbf{x}}^{(B)} + (1-\rho) \cdot \hat{\mathbf{x}}^{(I)}\right) \cdot \gamma + \beta,$$

$$\rho \leftarrow \mathrm{clip}_{[0,1]}\left(\rho - \eta \Delta \rho\right)$$

- Adaptive Layer-Instance Normalization (AdaLIN)

$$AdaLIN(a,\gamma,\beta) = \gamma \cdot (\rho \cdot \hat{a_I} + (1-\rho) \cdot \hat{a_L}) + \beta,$$

$$\hat{a_I} = \frac{a-\mu_I}{\sqrt{\sigma_I^2+\epsilon}}, \hat{a_L} = \frac{a-\mu_L}{\sqrt{\sigma_L^2+\epsilon}},$$

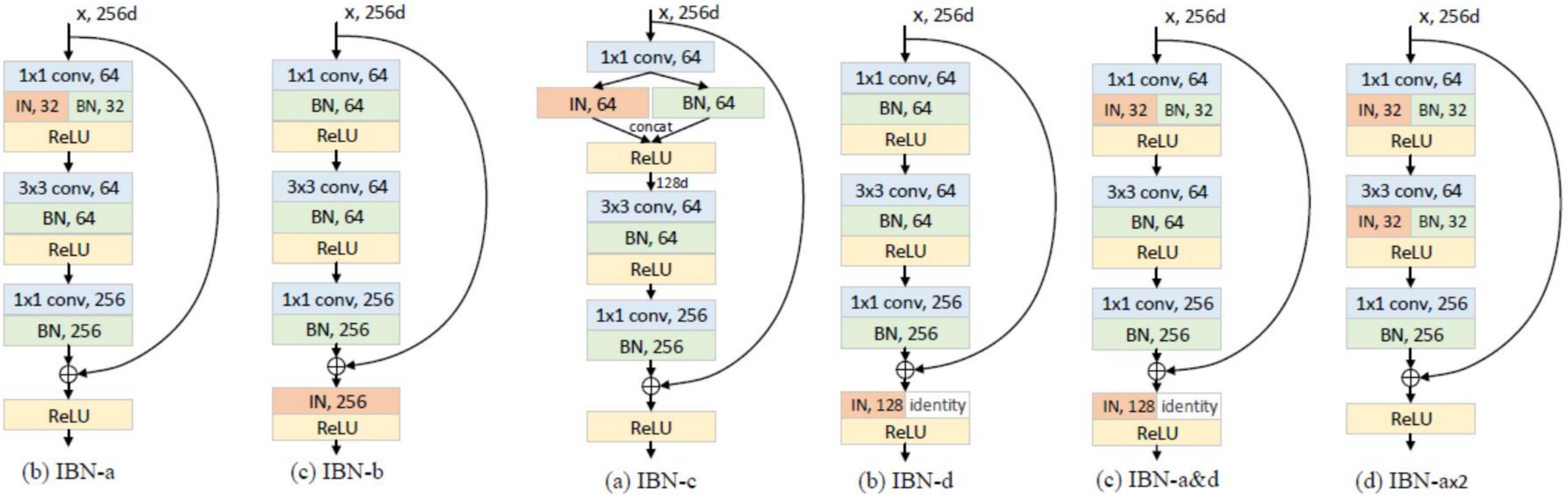
$$\rho \leftarrow clip_{[0,1]}(\rho - \tau\Delta\rho)$$

Generated by
a network

Attentive Normalization [Nam and Kim, NeurIPS 2018; Kim et al, ICLR 2020]

Combinational Normalization

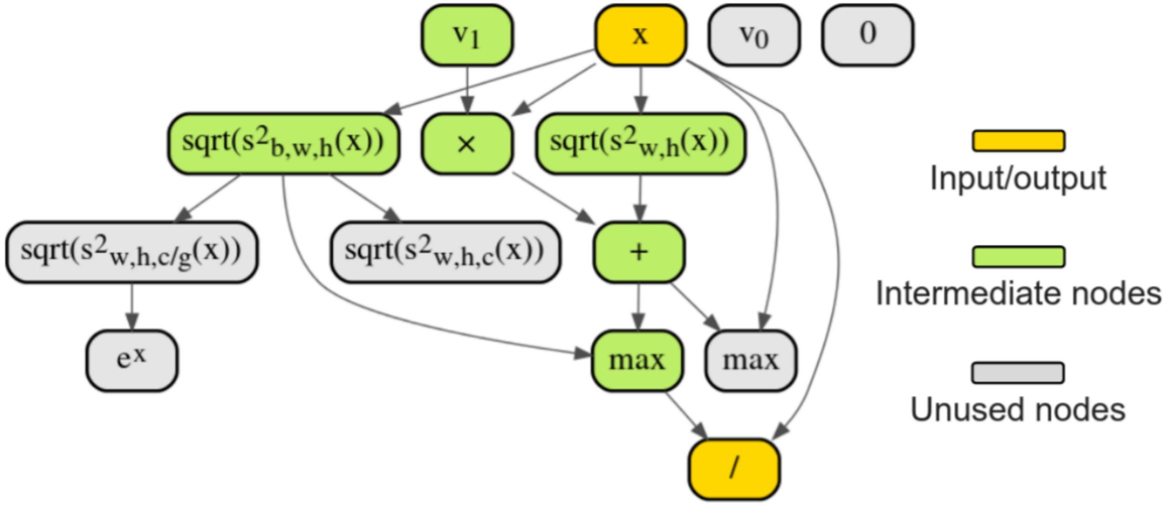
- Combination by design: IBN-Net



Two at Once: Enhancing Learning and Generalization Capacities via IBN-Net [Pan et al, ECCV 2018]

Normalization by Learning

- EvoNorm



BN-ReLU	$\max\left(\frac{x-\mu_{b,w,h}(x)}{\sqrt{s^2_{b,w,h}(x)}}\gamma+\beta, 0\right)$
EvoNorm-B0	$\frac{x}{\max\left(\sqrt{s^2_{b,w,h}(x)}, v_1 x+\sqrt{s^2_{w,h}(x)}\right)}\gamma+\beta$

Evolving Normalization-Activation Layers[Liu et al, NeurIPS 2020]

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    - Normalization Operation (NOP)
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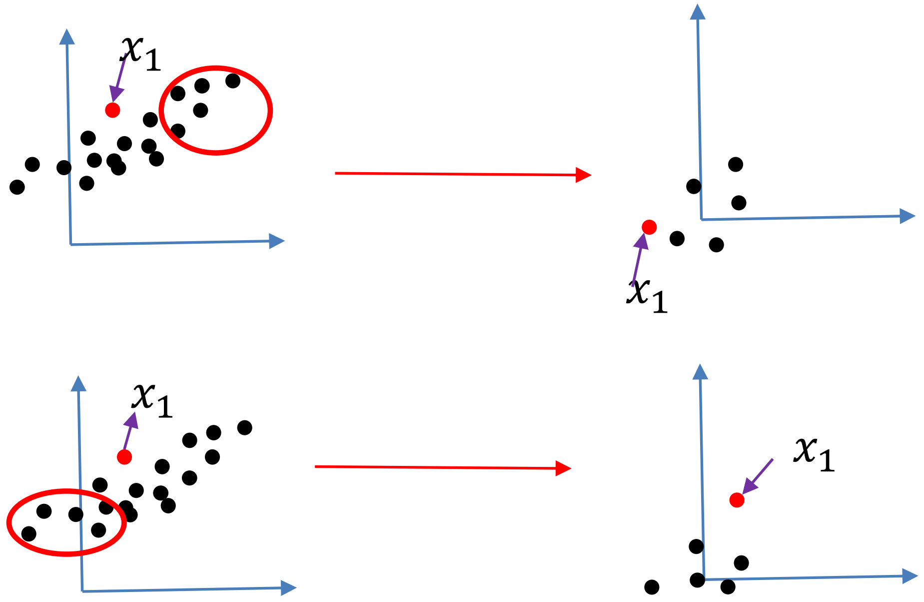
  - BN for more robust estimation

BN for More Robust Estimation

- Small batch size problem of BN
  - Large Training-test discrepancy
  - Large stochasticity during training



- Combining the population and mini-batch during training
  - [Dinh et al, ICLR 2016]

$$\begin{cases} \hat{u} = (1-\lambda)\hat{u} + \lambda u, \\ \hat{\sigma}^2 = (1-\lambda)\hat{\sigma}^2 + \lambda\sigma^2 \end{cases}$$

BN for More Robust Estimation

- Combining the population and mini-batch during training
  - Batch Re-normalization

$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m} x_i$$

$$\sigma_{\mathcal{B}} \leftarrow \sqrt{\epsilon + \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_{\mathcal{B}})^2}$$

$$r \leftarrow \texttt{stop\_gradient}\left(\text{clip}_{[1/r_{\max}, r_{\max}]}\left(\frac{\sigma_{\mathcal{B}}}{\sigma}\right)\right)$$

$$d \leftarrow \texttt{stop\_gradient}\left(\text{clip}_{[-d_{\max}, d_{\max}]}\left(\frac{\mu_{\mathcal{B}} - \mu}{\sigma}\right)\right)$$

$$\widehat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sigma_{\mathcal{B}}} \cdot r + d$$

$$y_i \leftarrow \gamma\, \widehat{x}_i + \beta$$

Update moving averages

$$\mu := \mu + \alpha(\mu_{\mathcal{B}} - \mu)$$
$$\sigma := \sigma + \alpha(\sigma_{\mathcal{B}} - \sigma)$$

Batch Renormalization: Towards Reducing Minibatch Dependence in Batch-Normalized Models[Ioffe, NeurIPS 2017]

BN for More Robust Estimation

- Normalization as Functions Combining Population Statistics
  - Online normalization [Chiley et al, NeurIPS 2019]
  - Towards stabilizing batch statistics in backward propagation of batch normalization [Yan et al, ICLR 2020]
  - PowerNorm: rethinking batch normalization in transformers [Shen et al, ICML 2020]
  - Momentum batch normalization for deep learning with small batch size [Yong et al, ECCV 2020]
  - Double forward propagation for memorized batch normalization [Guo et al, AAAI 2018]
  - Cross-iteration batch normalization [Yao et al, CVPR 2021]

BN for More Robust Estimation

- Robust inference methods for BN
  - Estimating population statistics after training
    - Ioffe and Szegedy, 2015
    - Luo et al, 2018
    - Wu et al, 2021
  - Estimating batch normalization statistics for evaluation
    - EvalNorm [Singh et al, ICCV 2019]
    - [Summers and Dinneen, ICLR 2020]

Outline

Introduction of Normalization Methods

- Normalize Activation
  - By population statistics
  - As functions
- Normalize Weight
- Normalize Gradient

Normalizing Weights

- The general idea: normalize the activation implicitly during training

- Normalization Propagation [Arpit et al 2016; Shekhovtsov et al, 2018]
  - Normalize input: 0-mean and unit variance
  - Assuming **W** is orthogonal
  - Derivate the nonlinear dynamic, e.g. Relu:

Remark 1. *(Post-ReLU distribution) Let* $X \sim \mathcal{N}(0,1)$ *and* $Y = \max(0, X)$. *Then* $\mathbb{E}[Y] = \frac{1}{\sqrt{2\pi}}$ *and* $\mathrm{var}(Y) = \frac{1}{2}\left(1 - \frac{1}{\pi}\right)$

- Deriving the dynamics of activation by designing non-linearity [Shang et al, 2017; Klambauer et al, 2017]

Weight Normalization

- Target BN's drawback:
  - Unstable for small mini batch size
  - RNN

- Express weight as new parameters

$$\mathbf{w} = \frac{g}{||\mathbf{v}||}\mathbf{v} \qquad\qquad y = \phi(\mathbf{w} \cdot \mathbf{x} + b)$$

- Decouple direction and length of weight vectors

Weight Normalization: A Simple Reparameterization to Accelerate Training of Deep Neural Networks [Salimans et al, NeurIPS 2016]

Centered Weight Normalization

- Motivated by initialization methods: zero-mean, stable variance
  [Glorot et al, AISTATS 2010; He et al, ICCV 2015]

- Constrained optimization problem:

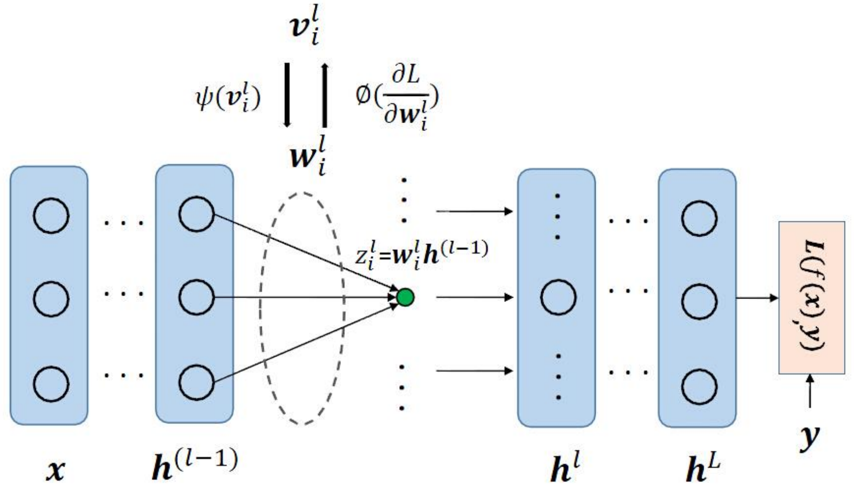
$$\theta^* = \arg\min_\theta \mathbb{E}_{(\mathbf{x},\mathbf{y})\in D}[\mathcal{L}(\mathbf{y}, f(\mathbf{x};\theta))]$$
$$s.t. \quad \mathbf{w}^T\mathbf{1} = 0 \; and \; \|\mathbf{w}\| = 1$$

- Solution by re-parameterization
  - Using proxy parameter v

$$\mathbf{w} = \frac{\mathbf{v} - \frac{1}{d}\mathbf{1}(\mathbf{1}^T\mathbf{v})}{\|\mathbf{v} - \frac{1}{d}\mathbf{1}(\mathbf{1}^T\mathbf{v})\|}$$

  - Gradient information:

$$\frac{\partial\mathcal{L}}{\partial\mathbf{v}} = \frac{1}{\|\hat{\mathbf{v}}\|}[\frac{\partial\mathcal{L}}{\partial\mathbf{w}} - (\frac{\partial\mathcal{L}}{\partial\mathbf{w}}\mathbf{w})\mathbf{w}^T - \frac{1}{d}(\frac{\partial\mathcal{L}}{\partial\mathbf{w}}\mathbf{1})\mathbf{1}^T]$$



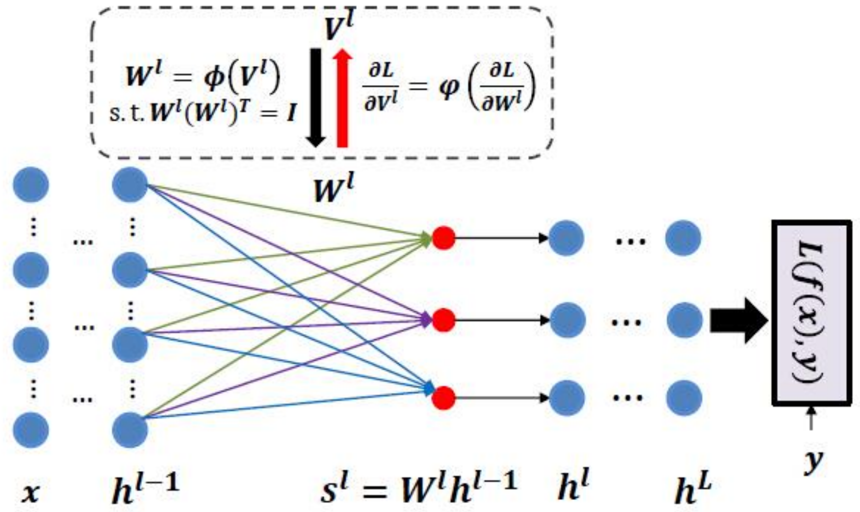
Centered Weight Normalization in Accelerating Training of Deep Neural Networks [Huang et al, ICCV 2017]

Orthogonal Weight Normalization

- Motivation: orthogonal initialization [Saxe et al, ICLR 2014; Mishkin et al, ICLR 2016]
  - Activation: $\| h \| = \| x \|$
  - Gradient: $\| \frac{\partial L}{\partial x} \| = \| \frac{\partial L}{\partial h} \|$
- Constrained optimization problem over multiple Stiefel manifold:

$$\theta^* = \arg\min_\theta \mathbb{E}_{(\mathbf{x},\mathbf{y})\in D}\left[\mathcal{L}\left(\mathbf{y}, f\left(\mathbf{x};\theta\right)\right)\right]$$
$$s.t. \quad \mathbf{W}^l \in \mathcal{O}_l^{n_l \times d_l}, l = 1, 2, ..., L$$

- Solution by re-parameterization



Orthogonal Weight Normalization: Solution to Optimization over Multiple Dependent Stiefel Manifolds in Deep Neural Networks [Huang et al, AAAI 2018]

Normalizing Weights

- Constraints with optimization:

$$\theta^* = \arg\min_\theta \mathbb{E}_{(\mathbf{x},\mathbf{y})\in D}[\mathcal{L}(\mathbf{y}, f(\mathbf{x};\theta))] \quad s.t. \quad \Upsilon(\boldsymbol{W}),$$

- Weight normalization:

$$\Upsilon(\boldsymbol{W}) = \{\|\boldsymbol{W}_i\| = 1, i = 1, ..., d_{out}\}$$

- Centered weight normalization/Scaled weight standardization:

$$\Upsilon(\boldsymbol{W}) = \{\boldsymbol{W}_i^T\mathbf{1} = 0 \,\&\, \|\boldsymbol{W}_i\| = 1, i = 1, ..., d_{out}\}$$

- Weight standardization:

$$\Upsilon(\boldsymbol{W}) = \{\boldsymbol{W}_i^T\mathbf{1} = 0 \,\&\, \|\boldsymbol{W}_i\| = \sqrt{d_{out}}, i = 1, ..., d_{out}\}$$

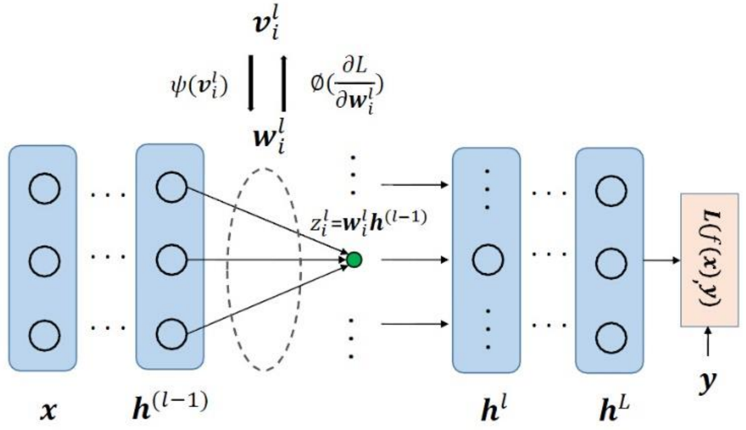
- Orthogonal weight normalization:

$$\Upsilon(\boldsymbol{W}) = \{\boldsymbol{W}\boldsymbol{W}^T = \boldsymbol{I}\}$$

[Salimans et al, NeurIPS 2016; Huang et al, ICCV 2017;Huang et al, AAAI 2018; Qiao et al, 2019; Brock et al, ICLR 2021]

Training with Constraints

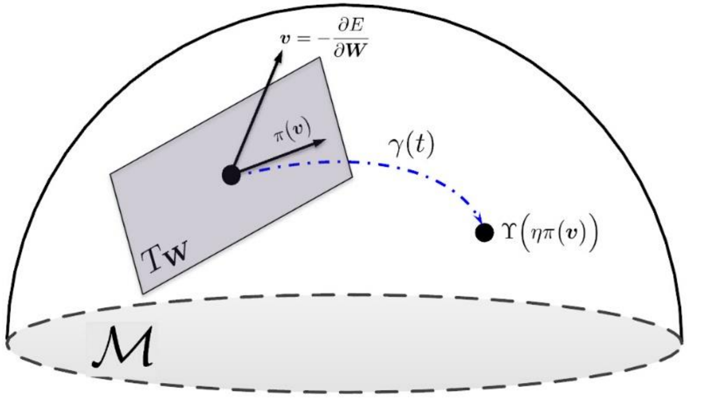
- Re-Parameterization



- Regularization with an extra penalty

$$\theta^* = \arg\min_\theta \mathbb{E}_{(\mathbf{x},\mathbf{y})\in D}\left[\mathcal{L}\left(\mathbf{y}, f\left(\mathbf{x};\theta\right)\right)\right] + \frac{\lambda}{2}\sum_{i=1}^{D} \|\mathbf{W}_l^T \mathbf{W}_l - \mathbf{I}\|_F^2$$

- Riemannian optimization



[Huang et al, ICCV 2017; Cisse et al, ICML 2017; Bansal et al; NeurIPS 2018; Ozay and Okatani, 2016]

Discussions with Normalizing Weights

- Advantages over activation normalizations
  – No extra cost during inference
  – Not sensitive to the batch size, compared to BN

- Disadvantages over activation normalizations
  – It is not stable, compared to activation normalizations
  – It needs to well design the gain parameters for satisfying criteria 1 (equivalent variance/distribution among layers).
  – The gain parameters depend on the network architectures [Huang et al 2017, Brock et al, 2021], thus it is more difficult to use in practice

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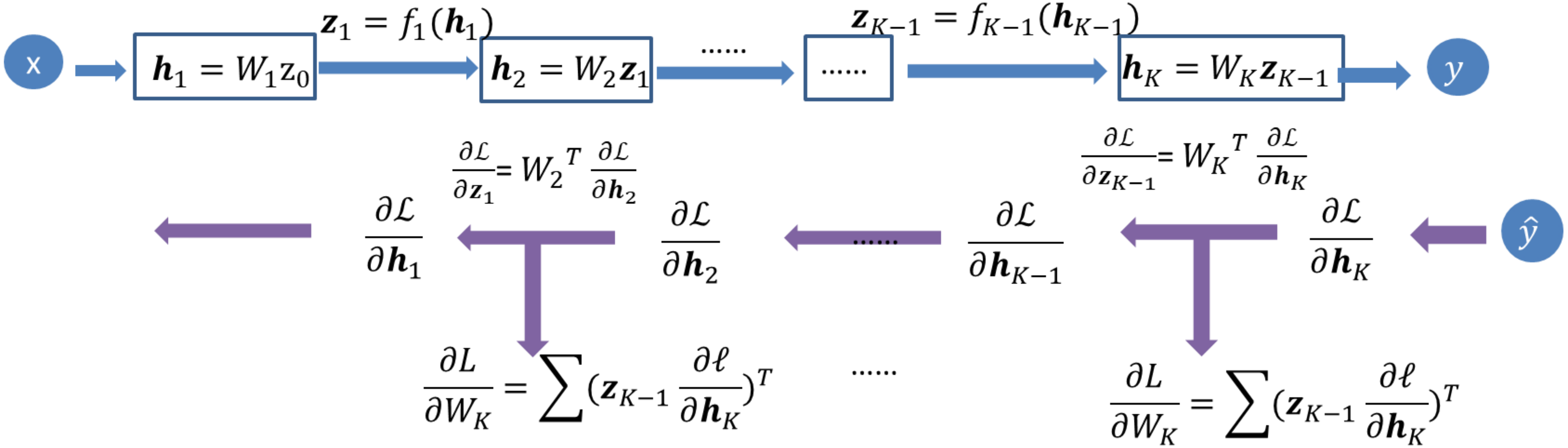
Normalizing Gradients



- Block-wise (layer-wise) gradient normalization [Yu et al, 2017]
  - $\widehat{\frac{\partial L}{\partial W_K}} = \frac{\frac{\partial L}{\partial W_K}}{\|\frac{\partial L}{\partial W_K}\|_2}$
  - $\widehat{\frac{\partial L}{\partial W_K}} = \alpha \frac{\frac{\partial L}{\partial W_K}\|W_K\|_2}{\|\frac{\partial L}{\partial W_K}\|_2}$ (adaptive for scale-invariant network)
- Layer-wise Adaptive Rate Scaling (LARS) [You et al 2017], for large batch training

Normalizing Gradients

- LAMB [You et al, 2020]
  - LARS + Adam, for large-batch BERT training

- LANS [Zheng et al, 2020]
  - Incorporate Nesterov’s Momentum into LAMB, for large-batch BERT training

- Gradient centralization [Yong et al, 2020]
  - $\widehat{\frac{\partial L}{\partial W_K}} = (\mathrm{I} - \boldsymbol{e}\boldsymbol{e}^T)\frac{\partial L}{\partial W_K}$

Q&A